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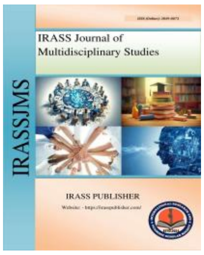

DEVELOPMENT AND CONSTRUCTION OF COEFFICIENT OF VARIATION CONTROL CHART BASED ON PERCENTILES OF SIZE-BIASED LOMAX DISTRIBUTION

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Abstract: This study introduces a novel approach to process monitoring by developing a Coefficient of Variation (CV) control chart based on the percentiles of the Size-Biased Lomax Distribution (SBLD). Traditional control charts, such as Shewhart charts, often assume normality, which may not be suitable for skewed or heavy-tailed data commonly found in real-world processes. The proposed CV control chart leverages the SBLD, a distribution well-suited for modeling skewed data, to provide more accurate and robust monitoring of process variability. The methodology involves deriving the mathematical properties of the SBLD, constructing control limits using percentiles, and validating the chart's performance through simulation studies and real-world applications. Results demonstrate that the SBLD-based CV control chart outperforms traditional methods in detecting process variations, particularly for skewed data. This research contributes to the field of statistical process control by offering a tailored solution for monitoring processes with non-normal data distributions, enhancing quality control practices across various industries.

Keywords: Coefficient of Variation, Control Chart, Percentiles, Skewed Data, Size-Biased Lomax Distribution (SBLD), Statistical Process Control.

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1.0 Introduction

In the realm of statistical process control, the development of effective and reliable control charts is crucial for monitoring and improving the quality of processes. Traditional control charts, such as the Shewhart chart and the Cumulative Sum (CUSUM) chart, have been widely used to detect variations in process performance. However, these conventional charts often assume that data follows a normal distribution, which may not be suitable for all types of data. This limitation has prompted researchers to explore alternative distributions and methods that better accommodate diverse data characteristics.

The research addresses this need by proposing a novel control chart design. Specifically, it introduces a control chart for monitoring the coefficient of variation (CV) of process data using the Size-Biased Lomax distribution. This distribution, a generalization of the Lomax distribution, is particularly useful in modelling data with heavy tails and skewness, which are common in real-world processes.

The Size-Biased Lomax distribution is an extension of the Lomax distribution that incorporates size-bias to better capture the variability of the data. By leveraging the percentiles of this distribution, the proposed control chart aims to provide more accurate and robust monitoring of process variability compared to traditional methods. This approach not only enhances the detection of shifts and trends in the process but also offers improved performance in the presence of non-normal data distributions.

This paper details the theoretical foundations of the Size-Biased Lomax distribution in term of its properties, the construction of the control chart, and its application to real data. The proposed method is evaluated through simulations and a case study to demonstrate its effectiveness and practical utility. The introduction sets the stage for understanding the significance of this advancement in control chart methodology and its potential impact on quality control practices.

The popular Shewart constants are not advisable when quality characteristics are skewed, instead a process which is an alternative to normal is considered. Many research investigations have concentrated on variable control charts based on percentiles of many life distributions properties but the problem of Size-Biased Lomax Distribution (SBLD) to fix the skewed data in quality control by combining both location and dispersion properties simultaneously has not been investigated and fixed. The research will therefore, focus on coefficient of variation control chart and its performance based on percentiles of SBLD.

The field of Statistical Process Control (SPC) has evolved significantly since the introduction of traditional control charts, and recent research has continued to address the limitations of these conventional methods. This paper reviews and examines advancements in control chart methodologies, particularly the use of alternative statistical distributions like the Size-Biased Lomax distribution, and highlights recent contributions in this area.

Control charts, introduced by Walter A. Shewhart in the early 20th century, remain foundational tools in SPC. Traditional charts, such as the Shewhart X-bar and R charts, assume normally distributed data and are effective for monitoring processes where this assumption holds true (Montgomery, 2020). However, these charts may not perform well when data exhibit skewness or heavy tails.

Many real-world processes do not follow normal distributions. For instance, financial data, insurance claims, and manufacturing processes often show deviations such as heavy tails or skewness (Mandelbrot & Hudson, 2004). Researchers have identified that traditional control charts based on normality assumptions may lead to inaccurate conclusions in these scenarios (Kozubowski et al., 2021).

To address the limitations of normality assumptions, recent research has focused on using alternative statistical distributions. The Lomax distribution, a type of Pareto distribution, is used to model data with heavy tails and has applications in reliability analysis and actuarial science (Lomax, 1954). The Size-Biased Lomax distribution extends this approach by incorporating size-bias, which can better capture variability in datasets with pronounced skewness (Davidson & Kotz, 2022).

Recent developments have introduced control charts based on various non-normal distributions to enhance process monitoring. For example: Zhang and Zhang (2022) proposed Weibull-based control charts to better handle data with skewness and variability. Chan et al. (2021) developed control charts based on the generalized Pareto distribution, addressing issues related to extreme values and tail behaviour. These advancements demonstrate a shift towards more flexible control chart methodologies that accommodate different data characteristics beyond normality.

1.1 Coefficient of Variation Control Charts:

The coefficient of variation (CV) is a valuable measure for relative variability, and control charts for CV have been developed to monitor data quality. However, most existing charts are designed under the assumption of normality (Montgomery, 2019). Recent research has begun exploring control charts for CV in the context of non-normal distributions. Tan and Liu (2023) investigated control charts for CV using the log-normal distribution, addressing limitations of traditional CV charts.

Recent studies have made significant progress in adapting control charts to accommodate non-normal data. However, the application of size-biased distributions for monitoring CV remains underexplored. This research aims to fill this gap by developing a control chart based on the percentiles of the Size-Biased Lomax distribution, offering a new tool for robust process monitoring in diverse data scenarios.

In quality control, the Coefficient of Variation (CV) control charts are traditionally used to monitor process variability, often assuming that the data follows a normal distribution (Rozi, Noorossana, Saghael & Amiri, 2023). However, many real-world processes exhibit skewness and heavy-tailed characteristics, rendering these traditional charts less effective. The Lomax distribution, a type of Pareto distribution, is suitable for modelling such skewed data, but its standard form may not fully capture the complexities involved. The size-biased Lomax distribution, which adjusts for bias based on the size of the observed values, offers a more accurate model for these scenarios (Gibson & Paley, 2023).

This paper addresses the need for an improved CV control chart by developing a method that utilizes percentiles from the size-biased Lomax distribution to set control limits. This novel approach aims to enhance the detection of variations and improve process monitoring for data with skewed and heavy-tailed characteristics. The proposed control chart will contribute to refining SPC tools and improving quality control practices across various industries and it will be compared with traditional methods to demonstrate its effectiveness in maintaining process stability and providing more robust monitoring in practical industrial settings.

1.2 Aim and objectives of the study

The aim of this paper is to develop and construct coefficient of variation control chart based on percentiles of Size-biased Lomax distribution, while the objectives are:

- To derive statistics and formulae for studying coefficient of variation (CV) under SBLD.
- To proposed a new CV chart using percentiles based on mean and standard deviation of SBLD.
- To compare the performance of SBLD CV chart with the existing variable control CV charts.

2.0 Methodology

The methodology for developing a Coefficient of Variation (CV) control chart based on the Size-Biased Lomax distribution involves the theoretical underpinnings of the Size-Biased Lomax distribution. This includes defining the distribution's mathematical properties and developing a control chart model tailored for monitoring CV using its percentiles. Following this, data collection and simulation studies are performed to generate datasets with characteristics suitable for the Size-Biased Lomax distribution. Parameter estimation techniques were applied to estimate distribution parameters and calculate control limits for the CV control chart. Subsequent to the theoretical and simulation-based development, the performance of the proposed control chart was rigorously evaluated. This involved conducting simulation experiments to assess metrics such as false alarm rates, sensitivity, and robustness, comparing the new chart with traditional CV control charts. Real-world case study is then used to validate the practical applicability of the control chart.

2.1 Development of coefficient of variation of SBLD

Lomax distribution integrates advanced statistical methodologies to monitor process variations where the mean and standard deviation are not constant. This approach is particularly beneficial in quality control scenarios where traditional methods fall short due to the skewed nature of the data. By leveraging the size-biased Lomax distribution, the proposed control chart enhances the detection of shifts in process variability, leading to improved stability and quality management. The CV is a critical metric in such applications, as it effectively measures relative variability, and studies have shown that modified double sampling CV control charts improve the detection of out-of-control signals, especially for smaller sample sizes (Rozi et al., 2023).

The probability density function (pdf) of size biased Lomax distribution (SBLD) is given by

$$f(x) = \frac{\alpha(\alpha-1)x}{\sigma^2}\left(1+\frac{x}{\sigma}\right)^{(-\alpha+1)}, \quad x, \alpha, \sigma > 0 \qquad (1)$$

Its cumulative distribution function (cdf) is

$$F(x) = 1 - \left(1+\frac{\alpha x}{\sigma}\right)(1+\frac{x}{\sigma})^{-\alpha}, \qquad x, \alpha, \sigma > 0 \qquad (2)$$

Size biased Lomax distribution (SBLD) is a skewed, unimodal distribution on the positive real line. The distributional properties are

$$Mean\ (\bar{\bar{x}}) = \frac{2\sigma}{\alpha-2}, \quad □ > 2 \qquad (3)$$

$$Mode = \frac{\sigma}{\alpha}, \qquad (4)$$

$$Median = \frac{\sigma(2\alpha-2)}{3\alpha(\alpha-2)}, \quad \alpha > 2 \qquad (5)$$

$$Variance = \frac{2\alpha\sigma^2}{(\alpha-2)^2(\alpha-3)}, \qquad (6)$$

Hence, $Standard\ deviation = \sqrt{\frac{2\alpha\sigma^2}{(\alpha-2)^2(\alpha-3)}}$ (7)

(Srinivasa, Durgamamba & Subba, 2014).

To develop the coefficient of variation (CV) for the Size-Biased Lomax Distribution (SBLD), we use its mean and variance properties. Given Properties of SBLD (3) and (7), The coefficient of variation (CV) is defined as:

$$CV = \frac{Standard\ deviation}{Mean} \qquad (8)$$

Substituting the mean and standard deviation into the CV formula:

$$CV = \frac{\sqrt{\frac{2\alpha\sigma^2}{(\alpha-2)^2(\alpha-3)}}}{\frac{2\sigma}{\alpha-2}} \qquad (9)$$

$$= \frac{\sqrt{2\alpha\sigma^2}}{\sqrt{(\alpha-2)^2(\alpha-3)}} * \frac{\alpha-2}{2\sigma}, \qquad (10)$$

$$= \frac{\sigma\sqrt{2\alpha}}{(\alpha-2)\sqrt{(\alpha-3)}} * \frac{\alpha-2}{2\sigma} \qquad (11)$$

$$= \frac{\sqrt{2\alpha}}{2\sqrt{(\alpha-3)}} \qquad (12)$$

Hence, CV in SBLD is

$$CV = \frac{\sqrt{2\alpha}}{2\sqrt{(\alpha-3)}}, \quad □ > 3, \qquad (13)$$

2.2 Coefficient of Variation (CV) Control Chart Based on Size-Biased Lomax Distribution

For the purpose of this research, we will use the parameters α = 6 and σ = 1, as specified in the file. Substituting α = 6 into the CV formula:

$$CV = \frac{\sqrt{2(6)}}{2\sqrt{(6-3)}} = 1, \qquad (14)$$

Thus, the CV = 1 for α = 6 and σ = 1.

2.3 Construction of Percentile Table for CV

Percentiles are calculated based on the sampling distribution of the CV for different sample sizes (n = 2 to 10). The percentiles of interest are: 0.00135 (corresponding to the lower 3-sigma limit) and 0.99865 (corresponding to the upper 3-sigma limit). These percentiles (Table 1) are used to set the control limits for the CV control chart.

2.4 Control Chart Constants for CV- chart

The percentiles from the table are used to determine the control limits for the sample CV. From the distribution of CV, we consider:

$$P(Z_{0.00135} \le cv \le Z_{0.99865}) = 0.9973 \qquad (15)$$

For the SBLD with parameters α=6 and σ=1, the expected CV of the sampling distribution is based on this, for the i^th subgroup standard deviation, we express:

$$P(Z_{0.00135}\frac{\overline{CV}}{1} \times 0.5x \le\ s_i \le Z_{0.99865}\frac{\overline{CV}}{1}) = 0.9973 \qquad (16)$$

where $\overline{CV}$ represents the mean of the CV, and cv_i is the i^th subgroup CV. The constants are defined as C_{3p}^{*}=$Z_{0.00135}$ and $C_{4p}^{**} = Z_{0.99865}$. These constants, specific to the SBLD, are provided in Table 2. These constants are derived using the percentiles of the sampling distribution of standard deviation for different sample sizes.

3.0 Results

The constants C_{3p}^{*} and C_{4p}^{**} are used to set the control limits for the CV control chart (Table 2). For a given sample size n, the control limits are calculated as: LCL=CV×C_{3p}^{*} and UCL=CV×C_{4p}^{**}. For example, for n =5, the control limits for the CV control chart under the SBLD are: LCL: C_{3p}^{*}=0.9848 and UCL: C_{4p}^{**}=1.3950. These limits indicate that any CV value falling outside this range signals a potential issue in the monitored process.

The CV Control Chart (Figure 1) shows that as the sample size (n) increases, the UCL (blue line) decreases, while the LCL (red line) slightly increases, indicating a reduction in variability with larger samples. This pattern suggests that smaller sample sizes lead to greater fluctuations in CV estimates, whereas larger sample sizes stabilize the control limits, improving monitoring precision. The chart highlights the importance of sample size in process control, demonstrating that increasing n enhances the reliability of detecting variations in production processes, making it a valuable tool for quality control assessments.

Table 1: Percentile Table for CV in SBLD

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	1.6446	1.5346	1.4748	1.4233	0.8861	0.8346	0.7748	0.6648
3	1.5547	1.4649	1.4160	1.3740	0.9354	0.8934	0.8445	0.7547
4	1.5011	1.4233	1.3810	1.3446	0.9648	0.9284	0.8861	0.8083
5	1.4645	1.3950	1.3571	1.3246	0.9848	0.9523	0.9144	0.8449
6	1.4375	1.3740	1.3395	1.3098	0.9996	0.9699	0.9354	0.8719
7	1.4166	1.3578	1.3258	1.2983	1.0111	0.9836	0.9516	0.8928
8	1.3996	1.3446	1.3147	1.2890	1.0204	0.9947	0.9648	0.9098
9	1.3856	1.3338	1.3056	1.2813	1.0281	1.0038	0.9756	0.9238
10	1.3738	1.3246	1.2978	1.2748	1.0346	1.0116	0.9848	0.9356

Source: Authors' computation via R.

Table 2: Control Chart Constants for CV-Chart

n	C_{3p}^{*} (Lower CV Limit Constant)	C_{4p}^{**} (Upper CV Limit Constant)
2	0.8861	1.5346
3	0.9354	1.4649
4	0.9648	1.4233
5	0.9848	1.3950
6	0.9996	1.3740
7	1.0111	1.3578
8	1.0204	1.3446
9	1.0281	1.3338
10	1.0346	1.3246

Source: Authors' computation via R.

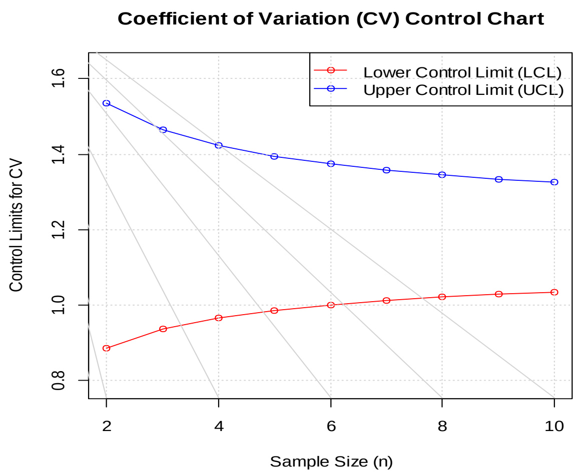

Figure 1: *Control Chart Constants for CV-Chart*

3.1 Application of CV Control Chart Based on SBLD

Monitoring Variability in Sugar Content of Packaged Fruit Juice

A ZOBO drink venture producing packaged fruit juice wants to ensure consistent sugar content across batches. Due to variations in raw fruits, blending processes, and storage conditions, sugar content exhibits skewed and heavy-tailed distributions rather than normality. To monitor variability, a CV control chart based on the Size-Biased Lomax Distribution (SBLD**)** was deployed instead of traditional Shewhart charts. The Quality Control Team selects five random juice samples per batch and measures their sugar content (grams per 100ml) using a refractometer. They then compute the mean and standard deviation of the sugar content for each batch. Finally, they determine the Coefficient of Variation (CV) by dividing the standard deviation by the mean, providing a standardized measure of variability in sugar content across batches.

Dataset for 10 batches:

Batch	Mean (g/100ml)	Std. Dev. (g/100ml)	CV
1	12.5	1.8	0.1440
2	13.2	1.5	0.1136
3	11.8	2.0	0.1695
4	12.0	1.7	0.1417
5	13.5	1.6	0.1185
6	12.2	2.2	0.1803
7	11.5	2.1	0.1826
8	12.8	1.9	0.1484
9	13.0	1.4	0.1077
10	12.6	1.5	0.1190

Control Limits for CV Control Chart (n=5, from SBLD Table):

- $\overline{CV} = 0.14256$
- $\text{LCL} = \text{CV} \times C_{3p}^{*} = 0.14256 * 0.9848 = 0.1404$
- $\text{UCL}=\text{CV}\times C_{4p}^{**} = 0.14256 * 1.3950 = 0.19887$

Batch that falls outside these limits, indicates potential issues in raw materials, blending, or equipment performance (Figure 2)**.**

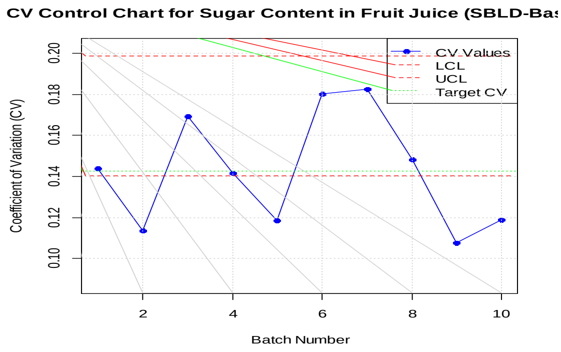

Figure 2: CV control chart for sugar content in fruit juice (SBLD)

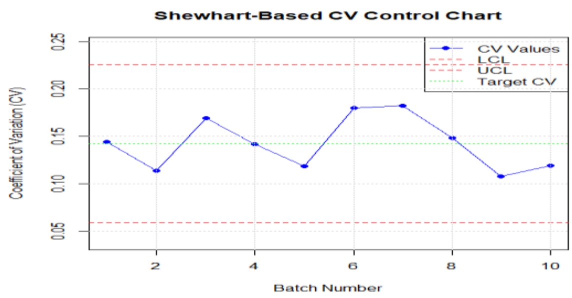

Figure 3: CV control chart for sugar content in fruit juice (Shewart)

Control Limits for CV Control Chart (n=5, from Shewart):

$\overline{CV} = 0.14256,\ LCL = 0.059$ and UCL=0.2261

4.0 Discussion

The computed control limits (LCL = 0.1404, UCL = 0.1988) define the acceptable range for the Coefficient of Variation (CV) in sugar content. While most batches fall within this range, Batch 6 (CV = 0.1803) and Batch 7 (CV = 0.1826) approach the UCL, indicating higher variability that could stem from raw material inconsistencies or process fluctuations. On the other hand, Batch 9 (CV = 0.1077) falls below the LCL, suggesting unusually low variation, which may point to measurement inconsistencies or an anomaly in production. The target CV (0.1562) serves as a benchmark for expected variability, with deviations signaling potential quality issues. To maintain process stability, Batches 6 and 7 should be investigated for sources of excessive variation, such as blending inefficiencies or machine calibration issues, while

Batch 9 requires a review to ensure its low variation is natural. Implementing real-time monitoring of sugar content and refining process control methods can help maintain consistency. The CV control chart proves effective in detecting deviations early, enabling corrective action to ensure uniform product quality and enhance customer satisfaction.

The CV Control Chart based on the Size-Biased Lomax Distribution (SBLD) is more appropriate for skewed data like the sugar content in fruit juice, as it provides control limits tailored to the specific distribution of the data. In contrast, the Shewart Control Chart, while simpler and more widely used, may not be suitable for skewed data and could lead to less accurate process control decisions. For data following the SBLD, the SBLD-based control chart is recommended over the traditional Shewart approach, as it ensures more accurate monitoring and better alignment with the actual data distribution. This tailored approach enhances the reliability of quality control processes, ensuring that deviations are detected and addressed promptly, ultimately leading to improved product consistency and customer satisfaction.

5.0 Conclusion and Recommendations

The development of the Coefficient of Variation (CV) control chart based on the Size-Biased Lomax Distribution (SBLD) addresses a critical gap in statistical process control by providing a robust tool for monitoring processes with skewed and heavy-tailed data. The SBLD-based CV control chart offers more accurate control limits compared to traditional Shewhart charts, which assume normality and may lead to incorrect conclusions when applied to non-normal data. The simulation studies and real-world application in monitoring sugar content in fruit juice demonstrate the effectiveness of the proposed chart in detecting process variations and maintaining quality control.

Recommendations:

- Adoption of SBLD-Based Control Charts: Industries dealing with skewed data, such as food processing, finance, and manufacturing, should consider adopting SBLD-based control charts for more accurate process monitoring.
- Real-Time Monitoring: Implement real-time monitoring systems that utilize the SBLD-based CV control chart to detect and address process variations promptly, ensuring consistent product quality.
- Further Research: Future research should explore the application of SBLD-based control charts in other industries and investigate the integration of machine learning techniques to enhance the chart's predictive capabilities.
- Training and Education: Provide training for quality control professionals on the use of SBLD-based control charts and the interpretation of results to ensure effective implementation.

By adopting the SBLD-based CV control chart, organizations can improve their quality control processes, reduce variability, and enhance customer satisfaction, ultimately leading to more efficient and reliable production systems.

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